PVP 19

Code: 19BS1301

II B.Tech - I Semester – Regular Examinations – MARCH 2021

ENGINEERING MATHEMATICS – III
(PDE Complex Variables and Transform Techniques)
(Common to CIVIL, EEE, ME, ECE)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 5 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each question carries 12 marks.
4. All parts of Question paper must be answered in one place

PART – A

1. a) If $L\{f(t)\} = \frac{1}{s(s^2+1)}$, find $L\{f(3t)\}$

b) Find the Fourier coefficient a_0 for $f(x) = x - x^2$ in $-\pi \le x \le \pi$.

c) Find the Fourier sine transform of $f(x) = \begin{cases} 1, & 0 \le x < 2 \\ 0, & x \ge 2 \end{cases}$

d) Show that $f(z) = z + 2\bar{z}$ is not analytic anywhere in the complex plane.

e) Classify the PDE: $3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - u = 0$

PART – B

UNIT – I

2. a) Evaluate $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$ 6 M

   b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$ 6 M

OR

3. a) Evaluate $L\{t\sin 3t\cos 2t\}$ 6 M

   b) Find the inverse Laplace transform of $\frac{4s+5}{(s-1)^2(s+2)}$ 6 M

UNIT – II

4. a) Find the Fourier series for the function 6 M

   $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$

   b) Obtain the half-range Fourier cosine series for $f(x)=x$ in $[0,2]$. 6 M

OR

5. a) Obtain the Fourier series for the function $f(x)=|x|$ in $-\pi\le x\le\pi$. 6 M

   b) Find the half-range Fourier sine series for $f(x)=x(\pi-x)$ in $0<x<\pi$. Hence deduce that 6 M

   $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots=\frac{\pi^3}{32}$

UNIT-III

6. a) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$. 6 M

b) Obtain Fourier cosine transform of 6 M

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

OR

7. a) Find the Fourier integral representation for 6 M

$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$.

b) Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence 6 M

show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} \; (m > 0)$

UNIT – IV

8. a) Show that $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \ne 0 \\ 0, & z = 0 \end{cases}$ is not analytic at $z = 0$ 6 M

although C-R equations are satisfied at that point

b) Find the analytic function whose real part is $y + e^x \cos y$ 6 M

OR

9. a) Evaluate $\oint_C \frac{z^3 - 2z + 1}{(z-i)^2} dz, \; C : |z| = 2$ by Cauchy's integral formula 6 M

b) Expand $f(z) = \frac{1}{(z-1)(z+3)}$ in Laurent's series for $1 < |z| < 3$ 6 M

UNIT – V

10. a) A tightly stretched string with fixed end points $x = 0$ & $x = l$ is initially in a position given by $y = y_0\ sin^3(\frac{\Pi x}{l})$. If it is released from rest from this position. Find the displacement $y(x,t)$. 6 M

b) Determine the solution of the initial boundary value problem 6 M

$$\frac{\partial y}{\partial t} = 16\frac{\partial^2 y}{\partial x^2}, 0 < x < l, t > 0,$$
$$y(0,t) = y(l,t) = 0, t > 0,$$
$$y(x,0) = (1-x)x, 0 < x < 1.$$

OR

11. a) Solve the following initial boundary value problem Obtain the solution of the initial boundary value problem 6 M

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0,$$
$$u(0,t) = u(\pi,t) = 0, t > 0,$$
$$u(x,0) = \sin x, 0 \le x \le \pi.$$

b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,$ where $u(x,0) = 6e^{-3x}$ by the method of separation of variables 6 M